

# Stability of hot tachyon gas

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## Abstract

We consider a tachyon gas that obeys Maxwell-Boltzmann statistics. The sound speed is always subluminal and it tends to the limiting minimum value  $c_s = 1/\sqrt{2}$  in non-relativistic gas (at low temperature), decreasing monotonously with the growth of temperature and attaining ultra-relativistic limit  $c_s = 1/\sqrt{3}$  at high temperature. The hot tachyon gas always satisfies the causality.

## 1 Introduction

The concept of tachyon fields plays significant role in the modern research, where they often appear in the field theory, cosmology, theory of branes and strings with various applications [1, 2, 3, 4, 5]. Tachyons, are commonly known as field instabilities whose energy spectrum is

$$\varepsilon_k = \sqrt{k^2 - m^2} \quad k > m \quad (1)$$

where  $m$  is the tachyon mass and relativistic units  $c = \hbar = 1$  are used.

A system of many tachyons can be studied in the frames of statistical mechanics [6, 7], and thermodynamical functions of ideal tachyon Fermi and Bose gases are calculated [8, 9].

We have recently considered a tachyon Fermi gas a zero temperature [10] and obtained a low-temperature expansion of its thermodynamical functions at finite temperature [11] as well as thermodynamical functions of fermionic thermal excitations [12]. All this analysis correspond to relatively low temperatures. At high temperature the Fermi distribution is reduced to the

Maxwell-Boltzmann distribution, and there is no problem to consider and ideal Maxwell-Boltzmann gas of tachyons [7]. However, the tachyon matter may occur unstable with respect to the causality condition [10]. So, the physical existence of hot tachyonic matter is not evident until we check

If we consider a system of many tachyons as continuous medium, we need to calculate the sound speed  $c_s$  and establish the range of parameters when the causality

$$c_s \leq 1 \quad (2)$$

is satisfied. Otherwise, the system will be unstable to hydrodynamical perturbations and will not be able to exist in nature. In the present paper we check the causality condition (2) for a Maxwell-Boltzmann gas of tachyons.

## 2 Thermodynamical functions

Consider an ideal gas of free tachyons with the energy spectrum  $\varepsilon_p$  (1) at finite temperature  $T$ . Let  $\mu$  be the chemical potential of this system. The particle number density  $n$ , energy density  $E$  and pressure  $P$  are determined by standard formulas [10]

$$n = \frac{\gamma}{2\pi^2} \int_m^\infty f_p p^2 dp \quad (3)$$

$$E = \frac{\gamma}{2\pi^2} \int_m^\infty f_p \varepsilon_p p^2 dp \quad (4)$$

$$P = \frac{\gamma}{6\pi^2} \int_m^\infty f_p p^3 \frac{\partial \varepsilon_p}{\partial p} d\varepsilon \quad (5)$$

where  $f_p$  is the distribution function. It is taken in the form of Maxwell-Boltzmann distribution function

$$f_p = \exp [(\mu - \varepsilon_p)/T] \quad (6)$$

for hot matter at high temperature. With dimensionless variables

$$x = \frac{\varepsilon_k}{T} \quad \beta = \frac{m}{T} \quad (7)$$

the thermodynamical functions of tachyon matter (3)- (5) are written in the form:

$$n = \frac{\gamma T^3}{2\pi^2} \exp\left(\frac{\mu}{T}\right) \int_0^\infty \sqrt{x^2 + \beta^2} x \exp(-x) dx \quad (8)$$

$$E = \frac{\gamma T^4}{2\pi^2} \exp\left(\frac{\mu}{T}\right) \int_0^\infty \sqrt{x^2 + \beta^2} x^2 \exp(-x) dx \quad (9)$$

$$P = \frac{\gamma T^4}{6\pi^2} \exp\left(\frac{\mu}{T}\right) \int_0^\infty \left(\sqrt{x^2 + \beta^2}\right)^3 \exp(-x) dx \quad (10)$$

Integrating (10) by parts, we immediately find that

$$P = nT \quad (11)$$

Hence

$$E = nT J\left(\frac{m}{T}\right) \quad (12)$$

where

$$J(\beta) = \frac{\int_0^\infty \sqrt{x^2 + \beta^2} x^2 \exp(-x) dx}{\int_0^\infty \sqrt{x^2 + \beta^2} x \exp(-x) dx} \quad (13)$$

For relativistic Maxwell-Boltzmann gas of subluminal massive particle the pressure and energy density are given by the same formulas (11) and (12) with the integral (13) replaced by

$$J(\beta) = \frac{\int_0^\infty \sqrt{p^2 + m^2} \exp\left(-\frac{\sqrt{p^2 + m^2}}{T}\right) p^2 dp}{\int_0^\infty \exp\left(-\frac{\sqrt{p^2 + m^2}}{T}\right) p dp} = \frac{\int_\beta^\infty \sqrt{x^2 - \beta^2} x^2 \exp(-x) dx}{\int_\beta^\infty \sqrt{x^2 - \beta^2} x \exp(-x) dx} \quad (14)$$

where  $x = \varepsilon_p/T = \sqrt{p^2 + m^2}/T$  and  $\beta$  is defined according to (7). In the ultra-relativistic limit ( $\beta \rightarrow 0$ ) both (13) and (14) tend to

$$J(\beta) \rightarrow 3 \quad (15)$$

that corresponds to ultra-relativistic equation of state  $P = E/3$ . Non-relativistic approximation of (14) correspond to  $\beta = m/T \gg 1$ , and it is no more than

$$J(\beta) \rightarrow \beta + \frac{3}{2} = \frac{m}{T} + \frac{3}{2} \quad (16)$$

while for (13) we have

$$J(\beta) = 2 + \frac{3}{2\beta^2} = 2 + \frac{3}{2} \frac{T^2}{m^2} \quad (17)$$

Expressions (12) and (16) imply that

$$E \rightarrow mn + \frac{3}{2}nT \quad (18)$$

for subluminal particles, while (12) and (17) imply

$$E \rightarrow 2nT \quad (19)$$

for tachyons. The ratio

$$w(\beta) = \frac{P}{E} = \frac{1}{J(\beta)} \quad (20)$$

at arbitrary  $\beta$  is given in Fig. 1.

### 3 Sound speed

The sound speed is defined as

$$c_s^2 = \left( \frac{\partial P}{\partial E} \right)_S \quad (21)$$

and we can calculate it for a tachyon gas if we apply the method of its calculation for an ideal gas of subluminal particles [13].

We assume that the first law of thermodynamics is valid for tachyons:

$$dU = -PdV + TdS \quad (22)$$

where

$$U = EV \quad (23)$$

is the internal energy,  $V$  is the volume and  $S$  is the entropy. The number of particles  $N = nV$  is conserved and does not depend on temperature, in contrast to the non-conserved number of thermal excitations [10]. Then, we immediately define the specific heat per particle

$$C_V = T \left( \frac{\partial (S/N)}{\partial T} \right)_V = \frac{\partial (U/N)}{\partial T} = \frac{\partial (E/n)}{\partial T} \quad (24)$$

that, in the light of (12) implies

$$C_V = \frac{d(JT)}{dT} = J(\beta) - \beta J'(\beta) \quad (25)$$

where

$$J'(\beta) = \frac{dJ}{d\beta} \quad (26)$$

and integral  $J$  is defined by (14) and by (13) for massive subluminal particles and for tachyons, respectively.

Equation (22) yields

$$\left( \frac{\partial U}{\partial V} \right)_S = -P \quad (27)$$

and

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial S}{\partial V} \right)_T - P \quad (28)$$

that in the light of (12) and (23) implies

$$\left( \frac{\partial S}{\partial V} \right)_T = \frac{P}{T} \quad (29)$$

Looking for adiabatic at  $S = \text{const}$  in the form

$$PV^\Gamma = \text{const} \quad (30)$$

we find from (11) that

$$\Gamma = 1 - \frac{V}{T} \left( \frac{\partial T}{\partial V} \right)_S \quad (31)$$

Making transformation of derivatives [14], we have

$$\left( \frac{\partial T}{\partial V} \right)_S = \frac{\partial (T, S)}{\partial (V, S)} = \frac{\partial (T, S)}{\partial (T, V)} \frac{\partial (T, V)}{\partial (V, S)} = - \left( \frac{\partial S}{\partial V} \right)_T \frac{T}{C_V} \quad (32)$$

Substituting (29) and (32) in (31) we obtain

$$\Gamma = 1 + \frac{1}{C_V} \quad (33)$$

In the light of (22), (23) and (27), equation (21) implies

$$\left(\frac{\partial P}{\partial E}\right)_s = \left(\frac{\partial P}{\partial (U/V)}\right)_s = \frac{\Gamma}{1 + U/(PV)} = \frac{\Gamma}{1 + E/P} \quad (34)$$

Substituting (12), (11), (25) and (33) in (34), we have

$$c_s^2 = \left\{ 1 + \left[ \frac{d(TJ)}{dT} \right]^{-1} \right\} \frac{1}{1 + J} = \left( 1 + \frac{1}{J - \beta J'} \right) \frac{1}{1 + J} \quad (35)$$

It is the sound speed in ideal gas with the Maxwell-Boltzmann distribution (6).

Formula (35) for non-relativistic subluminal particles (16) it yields

$$c_s^2 = \frac{5}{3} \frac{T}{m} \quad (36)$$

while for non-relativistic tachyons (17) the sound speed is

$$c_s^2 = \frac{1}{2} \quad (37)$$

and for ultra-relativistic material (15) the sound speed tends to the same limit

$$c_s^2 = \frac{1}{3} \quad (38)$$

The sound speed in tachyon gas at arbitrary temperature is shown in Fig. 2. It is clear that the causality condition (2) is always satisfied.

## 4 Conclusion

At high temperature an ideal gas of tachyons obey the Maxwell-Boltzmann distribution (6). Its pressure  $P = nT$  (11) is given by the same formula both for tachyons and ordinary particles, while the energy density is given by formula  $E = nTJ(T)$  (12), where integral  $J$  for tachyons (13) differs from that

(14) for subluminal particles. The tachyonic specific heat is calculated by formula (25) and it is given in Fig. 3. The tachyonic specific heat always exceeds the specific heat of ordinary gas, as well as the specific heat of tachyonic thermal excitations exceeds the specific heat of subluminal thermal excitations [12]. However, the specific heat of hot tachyon gas reveals anomalous decrease with temperature only when  $T > T_m \simeq 1.13m$  ( $\beta < 0.885$ ), while at  $T < T_m$  the specific heat is growing with temperature.

The sound speed is calculated by formula (35), it depends only on temperature  $T$ , and this dependence is given in Fig. 2. The sound speed in hot tachyon gas (35) is always subluminal although its behavior differs from that in the ordinary massive gas (Fig. 2). It implies that the causality (2) is automatically satisfied and the hot tachyon gas is stable in the whole range of densities and temperatures, in contrast to the tachyon Fermi gas at zero temperature [10] and tachyonic excitations [12]. Therefore, the hot tachyon may have free surface where  $P \rightarrow 0$  and may form compact self-gravitating objects.

The pressure and density of hot tachyon always obey inequality  $P < E/2$  (Fig. 1), while a cold tachyon Fermi gas [10] and a Fermi gas of tachyonic thermal excitations [12] can be 'hyperstiff'  $P > E$ . However, the 'stiffness' of hot tachyon Fermi gas depends only on temperature and does not depend on density, and the ratio  $P/E$  increases when the temperature decreases (Fig. 1), that bears resemblance with the gas of tachyonic thermal excitations [12] and never occurs in the ordinary relativistic gas of subluminal particles. This strange behavior may play important role in calculation of stellar models with tachyon content.

Formula for the sound speed (35) is universal and it can be developed for any ideal gas whose pressure is given by proportionality  $P \sim nT$ . For exotic matter this problem deserves special consideration.

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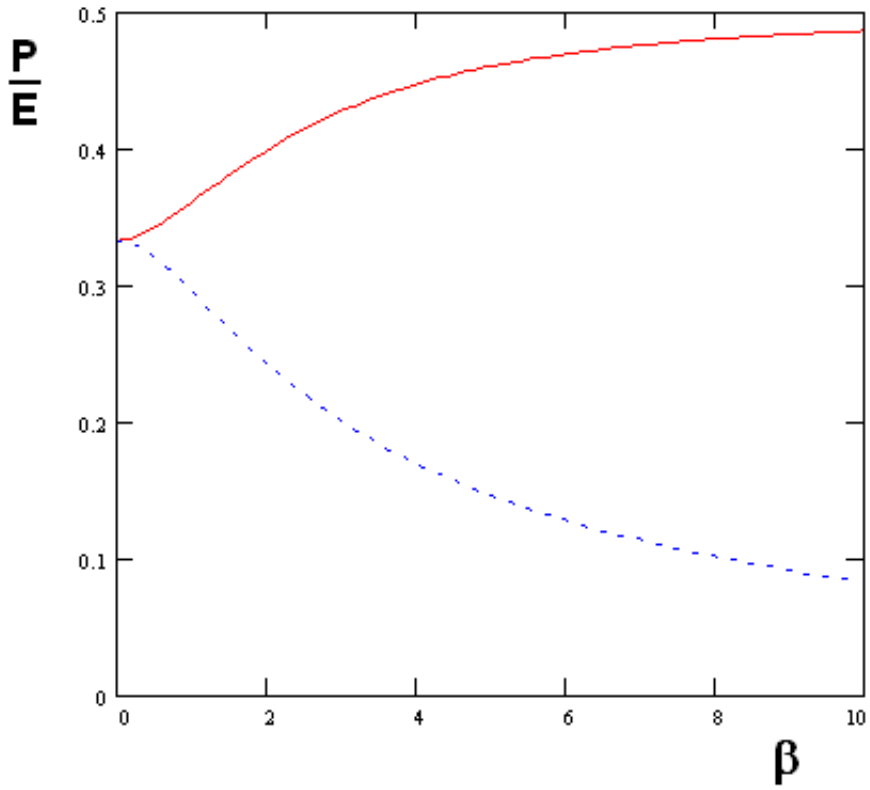
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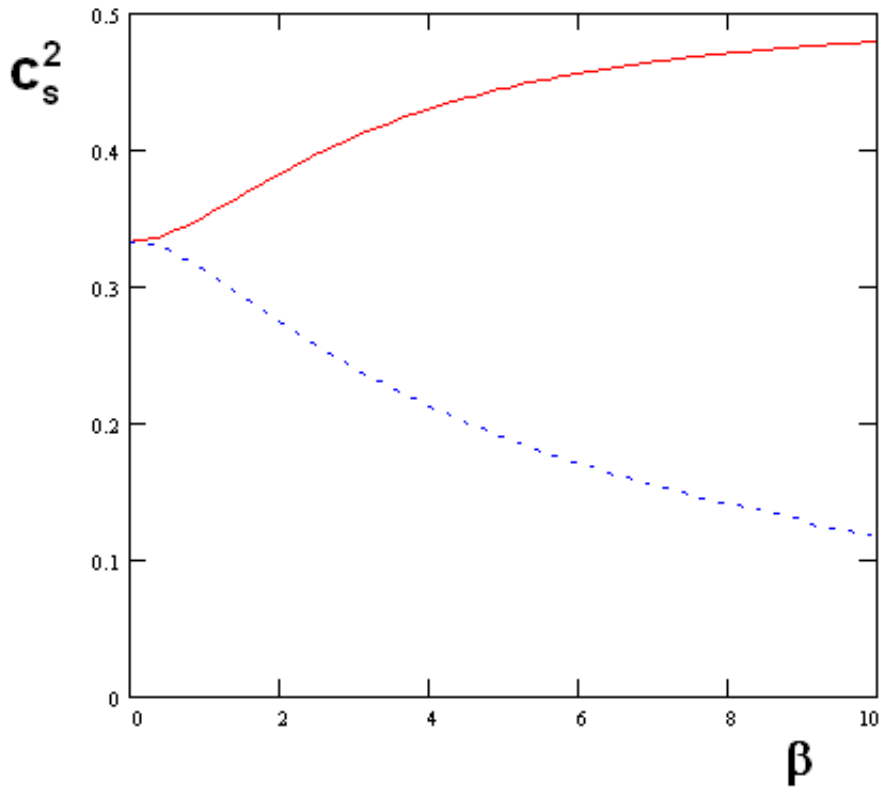


Figure 1: Ration of pressure to energy density  $P/E$  vs inverse temperature  $\beta = m/T$  for tachyons (solid line) and subluminal particles (dashed line).



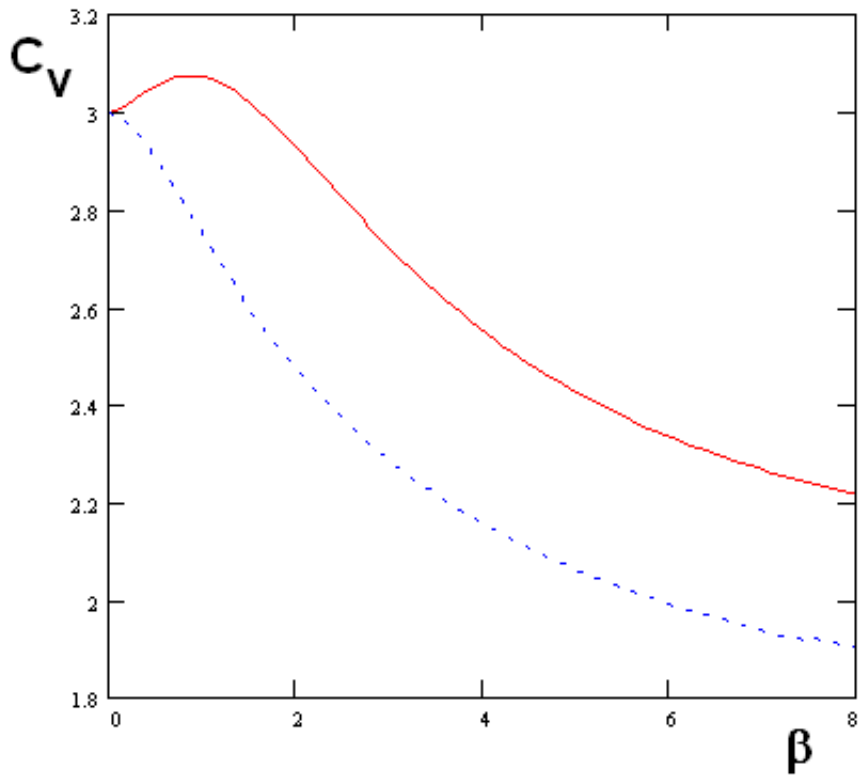
For non-relativistic tachyons ( $\beta \gg 1$ ) and  $P = E/2$ .

Figure 2: Sound speed  $c_s^2$  vs inverse temperature  $\beta = m/T$  for tachyons (solid line) and subluminal particles (dashed line).



For non-relativistic tachyons  $c_s^2 = 1/2$ .

Figure 3: Specific heat per particle  $C_V$  vs inverse temperature  $\beta = m/T$  for tachyons (solid line) and subluminal particles (dashed line).



For non-relativistic tachyons  $C_V = 2$  rather than  $3/2$ .